Walrasian Equilibrium

Disaggregated treatment of individual behavior, and behavior that need not be single-valued

Our approach to studying equilibrium so far has depended upon individuals' demand functions being single-valued and defined for all price-lists. This enabled us to add up (or aggregate) the individual demand functions to obtain an aggregate, or market, demand function, which would also be single-valued and defined for all price-lists. This reliance on the market demand function fails to cover a lot of completely non-pathological cases, such as Cobb-Douglas preferences or linear preferences.

The definition of market equilibrium we're about to give still requires that all markets clear. But by also including the individuals' choices as an explicit element of the equilibrium, we can deal with situations in which some price-lists leave some individuals indifferent among many available choices, as well as situations in which some price-lists leave some individuals with no optimal choice at all. But most important, framing things explicitly in terms of individual behavior will help us understand a much broader range of phenomena.

Definition: Let $E = \big((u^i, \mathring{\mathbf{x}}^i)\big)_1^n$ be an economy made up of n consumers $(u^1, \mathring{\mathbf{x}}^1)$, $\ldots$, $(u^n, \mathring{\mathbf{x}}^n)$. A *Walrasian equilibrium* of E is a pair $(\widehat{\mathbf{p}}, \widehat{\mathbf{x}}) \in \mathbb{R}_+^l \times \mathbb{R}_+^{nl}$ that satisfies

(1) $\forall i \in \{1, \ldots, n\}$: $\widehat{\mathbf{x}}^i$ maximizes u^i on the budget set

$$\mathcal{B}(\widehat{\mathbf{p}}, \mathring{\mathbf{x}}^i) := \{\, \mathbf{x}^i \in \mathbb{R}_+^l \mid \widehat{\mathbf{p}} \cdot \mathbf{x}^i \leqq \widehat{\mathbf{p}} \cdot \mathring{\mathbf{x}}^i \,\},$$

and

(2) $\forall k \in \{1, \ldots, l\}$: $\sum_{i=1}^n (\widehat{x}_k^i - \mathring{x}_k^i) \leqq 0$ and $\sum_{i=1}^n (\widehat{x}_k^i - \mathring{x}_k^i) = 0$ if $\widehat{p}_k > 0$.

Note that in this definition an equilibrium consists of both a price-list $\widehat{\mathbf{p}}$ *and* an allocation $\widehat{\mathbf{x}} = (\widehat{\mathbf{x}}^i)_1^n = (\widehat{\mathbf{x}}^1, \ldots, \widehat{\mathbf{x}}^n)$ that specifies the bundle each consumer receives.

Condition (1) says that at $\widehat{\mathbf{p}}$ the bundles $\widehat{\mathbf{x}}^1, \ldots, \widehat{\mathbf{x}}^n$ are consistent with the individual consumers' demand correspondences — *i.e.*, $\widehat{\mathbf{x}}^i \in D^i(\widehat{\mathbf{p}})$ for each i.

Condition (2) says that all markets clear: no good is in excess demand, and any good in excess supply has a zero price.